Econometric Modeling in Time Series: A Case Study in the Forward Market

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Time Series

The foundation of time series analysis is stationarity.

- Strictly stationary
  a) $E(Y_t) = \mu$, which is constant.
  b) Joint distribution $(Y_{t_1+t}, \cdots, Y_{t_k+t})$ is invariant under time shift.
- Weakly stationary / Covariance stationary
  a) $E(Y_t) = \mu$, which is constant
  b) Covariance $Cov(Y_t, Y_{t-l}) = \gamma_l$, which only depends on l
- Unit-root Non-stationary
  $E(Y_t)$ is not fixed.

Figure 1: British Pound 20 Years Daily Spot Rate

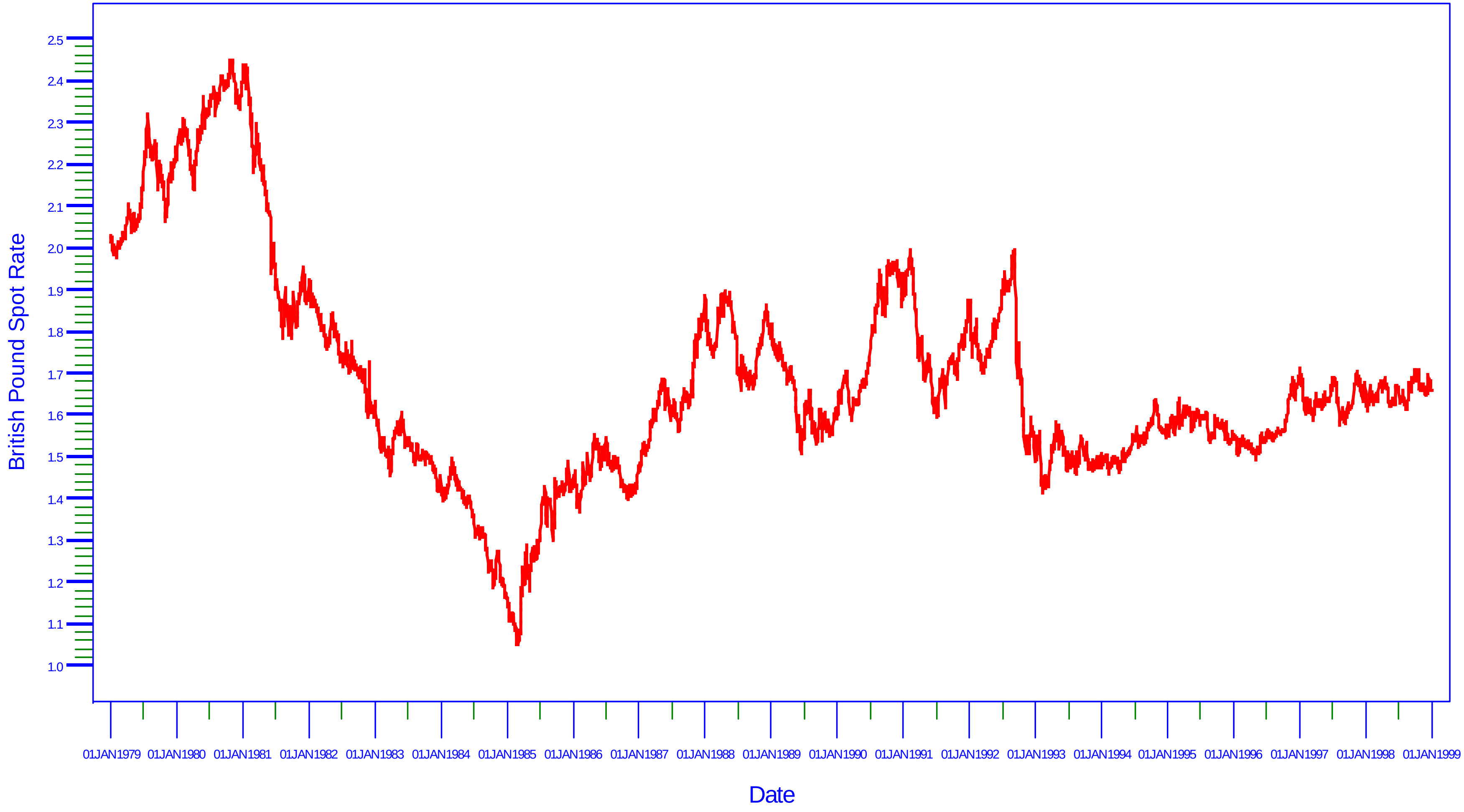
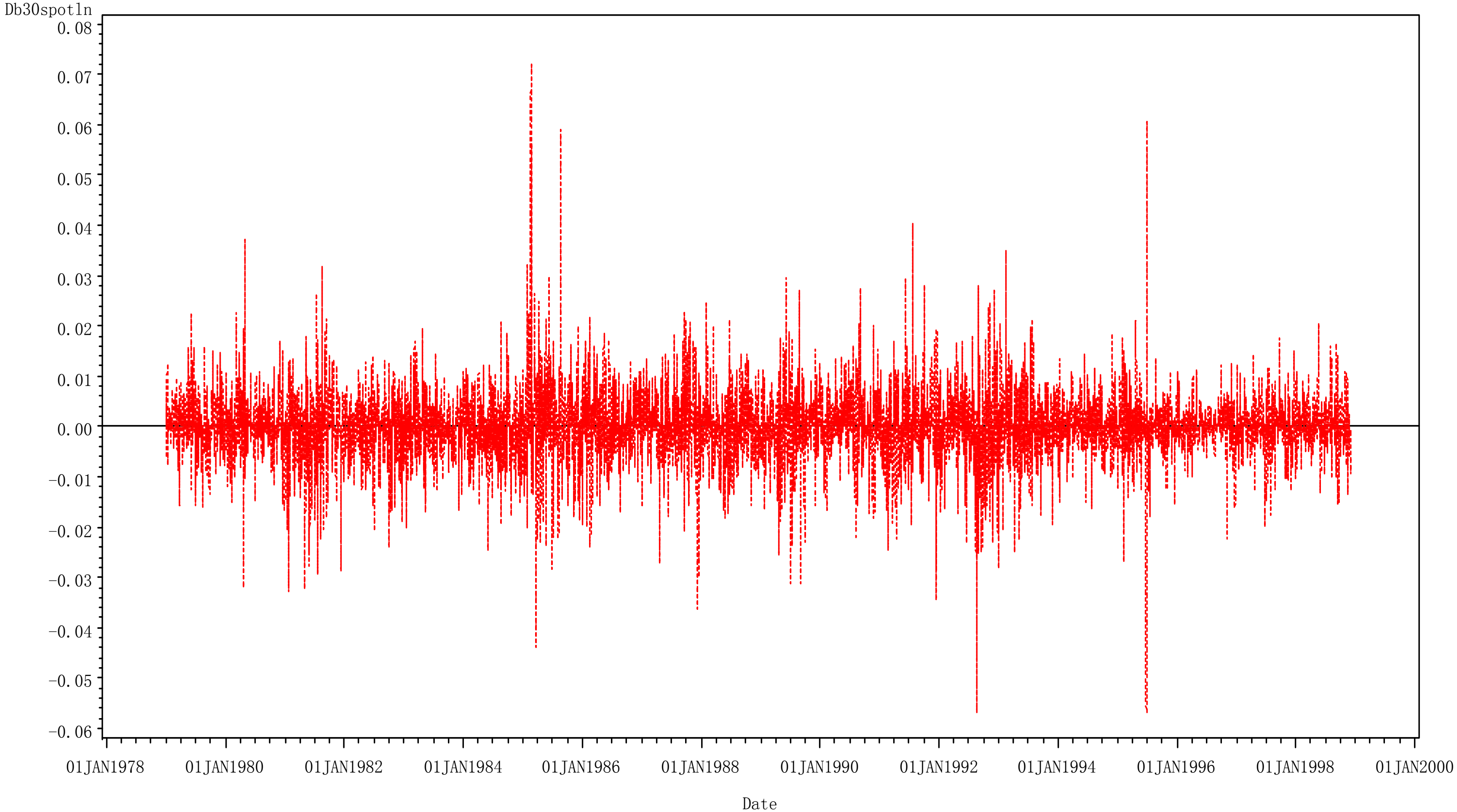


Figure 2: First Difference British Pound 20 Years Daily Spot Rate



AR Model

- AR(p) model form

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + \varepsilon_t$$

$$(Y_t - \mu) = \phi_1 (Y_{t-1} - \mu) + \cdots + \phi_p (Y_{t-p} - \mu) + \varepsilon_t \ \ Or \ \ (1 - \phi_1 B - \cdots - \phi_p B^p)(Y_t - \mu) = \varepsilon_t$$

- Property

$$E(Y_t) = \frac{\phi_0}{1 - \phi_1 - \cdots - \phi_p}$$

$$x^p - \phi_1 x^{p-1} - \phi_2 x^{p-2} - \cdots - \phi_p = 0$$

MA Model

- MA(q) model form

$$Y_t = c_0 + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q}$$

$$Y_t = c_0 + \left(1 - \theta_1 B - \cdots - \theta_q B^q\right)\varepsilon_t$$

- Property

$$E\left(Y_t\right) = c_0$$

$$V\left(Y_t\right) = \left(1 + \theta_1^2 + \cdots + \theta_q^2\right)\sigma_\varepsilon^2$$

Econometric Means

- $E(Y_t)$ has an important long-term implication for time series.
- For AR polynomials, the real-valued characteristic root shows an exponentially decaying feature. The complex characteristic roots imply the existing stochastic cycles, which could be approximately calculated by

$$cycle = \frac{360^0}{\cos^{-1}\left[\phi_m / 2\sqrt{-\phi_n}\right]}$$

- The effect of a random shock would disappear for q time periods as MA(q).

ARMA Models

- ARMA(p,q) model form

$$Y_t = \phi_0 + \sum_{i=1}^{p} \phi_i Y_{t-i} + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

$$\left(1 - \phi_1 B - \cdots - \phi_p B^p\right) Y_t = \phi_0 + \left(1 - \theta_1 B - \cdots - \theta_q B^q\right) \varepsilon_t$$

- Property

$$E\left(Y_t\right) = \phi_0 / \left(1 - \phi_1 - \cdots - \phi_p\right)$$

Neither ACF nor PACF does not cut off at any finite lag.

- SAS code

Four stages of ARMA modeling (Identification, Estimation, Model Checking and Forecasting).

p or & q order identification

	ACF	PACF	IACF	ESACF	SCAN	MINIC
AR(p)	-	X	X	X	X	X
MA(q)	X	-	-	X	X	X
ARMA(p,q)	-	-	-	X	X	X

```
proc arima data=path.yx30;
identify var=y_b esacf p=(0:10) q=(0:30) scan minic;
```

Estimation, Model Checking and Forecasting

- Estimation

```
estimate P=1 q=3 method=ml;
```

- Model Checking

Ljung_Box Q statistic

- Forecasting

```
forecast lead=5 out=path.out id=date interval=day;
run;
```

Regression Model with Time Series Errors

- Durbin-Watson Test

```
proc reg data=forward;
     model y = x / dw;
run;
```

- Regression model with AR(m)-GARCH(p,q) errors

```
proc autoreg data=stock;
     model y = x / nlag=3
     garch = (p=1, q=1);
     output out=result predicted=forecast lcl=l ucl=u;
run;
```

- Regression model with ARMA(p,q) errors

```
proc arima data=path.yx30 ;
     identify var=y_b crosscorr=(x_b) noprint;
     estimate input=(x_b) p=0 q=22 method=ml;
run;
```

Forward Market Efficiency

- Most literatures claim an empirical puzzle about the relationship between forward and spot foreign exchange rates.
- Unbiased Forward Rate Hypothesis (UFRH) got evidence from the empirical regression on future spot rate with forward rate. The regression parameter beta is always significantly close to one.

$$s_{t+k} = \alpha + \beta f_{t,t+k} + \mu_{t+k}$$

- On the other hand, the forward rate premium is often negatively correlated with subsequent changes in the spot rate at the significance level by monthly data. Fama (1984), Chiang (1988), Barnhart (1991, 1999), Hai (1997) and Roll (2000).

$$s_{t+k} - s_t = \alpha + \beta\left(f_{t,t+k} - s_t\right) + \zeta_{t+k} \qquad (7)$$

- Both popular results contradicted with each other.

Econometric Specification

- There is no serial correlation problem for the monthly data, which fits the SUR model.
- By the daily data, equation 7 will be the regression with ARMA errors.
- How to identify the order of ARMA errors?

✓ Two-step identification

run regression and save error, then identify the error by ESACF.

✓ Modeling

based on econometrics method, build model to identify the error structure.

ESACF Identification

p																q															
0	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	o	o	o	o	o	o	o	o	o	o	o	o
1	x	x	x	o	x	x	x	x	x	x	x	o	x	x	x	x	x	x	o	o	x	x	x	o	o	o	o	o	o	x	o
2	x	x	x	o	o	o	o	o	x	o	o	o	x	o	o	o	o	x	x	o	o	x	x	o	o	o	o	o	o	o	o
3	x	x	x	o	o	x	o	o	o	o	o	o	x	o	o	o	o	o	o	o	o	x	x	x	o	o	o	o	o	o	o
4	o	x	x	x	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	x	x	x	x	o	o	o	o	o	o
5	o	x	x	x	x	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	x	x	o	x	o	o	o	o	o	o
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30

Modeling ARMA Error and Attenuation Hypothesis

- Moving Average Structure

$$f_{t,t+k}^{theoretic} = E\left(s_{t+k} \middle| \Omega_t\right)$$

$$f_{t,t+k}^{theoretic} - s_t = E\left(s_{t+k} - s_t \middle| \Omega_t\right)$$

$$s_{t+1} - s_t = \epsilon_{t+1} + \sum_{j=1}^{\infty} \theta_j \epsilon_{t+1-j}$$

$$\xi_{t+k} = \sum_{i=1}^{k} \left(\varepsilon_{t+i} + \sum_{j=1}^{i-1} \theta_j \varepsilon_{t+i-j} \right) - E\left(\sum_{i=1}^{k} \left(\varepsilon_{t+i} + \sum_{j=1}^{i-1} \theta_j \varepsilon_{t+i-j} \right) \middle| \Omega_t \right)$$

- Attenuation

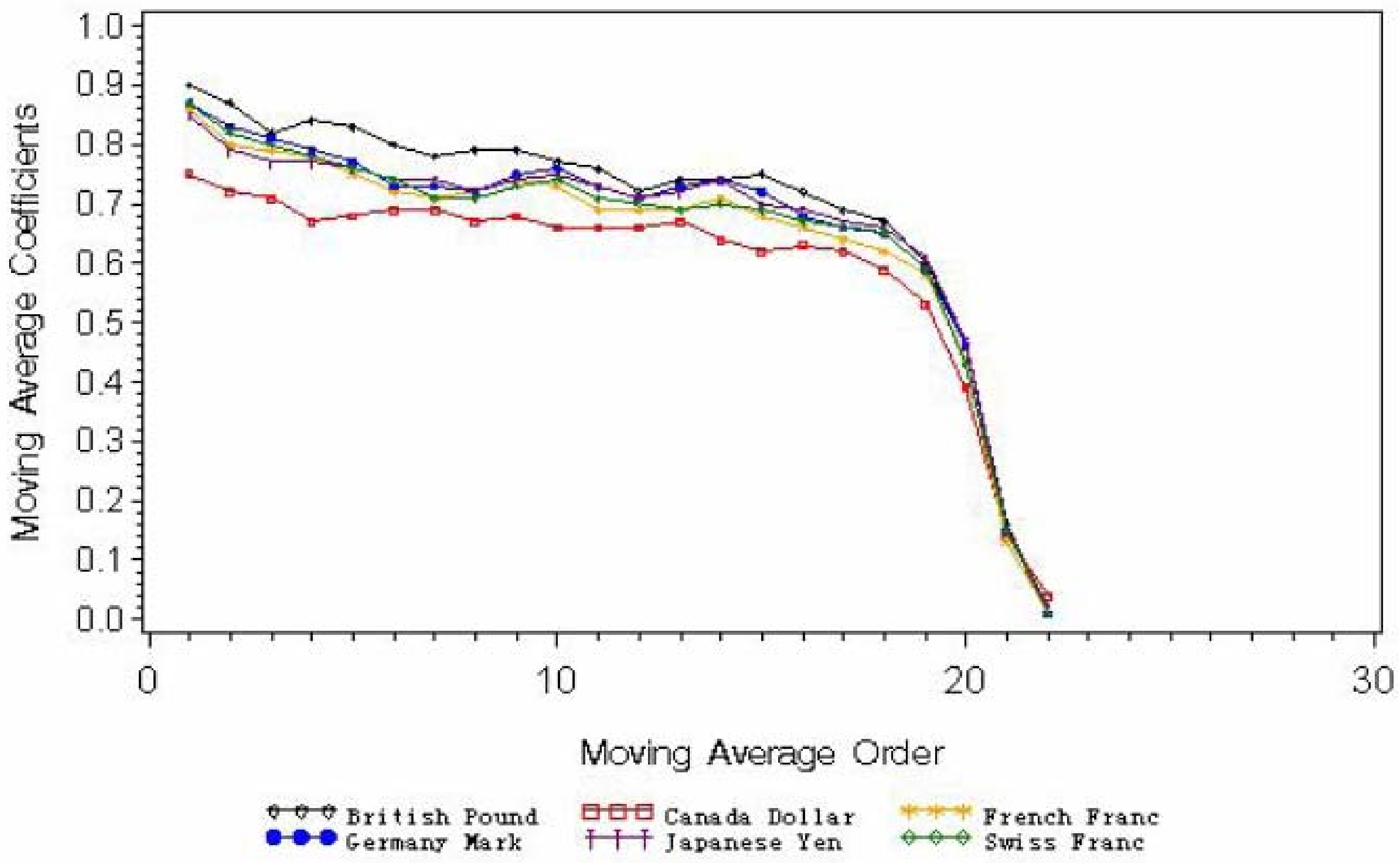
$$\beta = \frac{\frac{1}{n} \sum_{i=1}^{n} \left(\left(f_{t,t+k}^{theoretic} - s_t \right) + \gamma \right) \left(\left(f_{t,t+k}^{theoretic} - s_t \right) + u_{t+k} + \sum_{j=1}^{l} \eta_j u_{t+k-j} \right)}{\frac{1}{n} \sum_{i=1}^{n} \left(\left(f_{t,t+k}^{theoretic} - s_t \right) + \gamma \right)^2}$$

Estimation Results

	09/06/1977-12/01/1998 for 30-Day Forward Rate								
Country	α	t-ratio	β	t-ratio	(p, q)	Q_{24}	p	Q_{30}	p
Britain	0.00	0.39	0.40	6.40**	(0,22)	5.28	0.07	13.08	0.11
Canada	-0.00	-1.04	0.56	21.29**	(0,22)	4.54	0.10	13.85	0.09
France	-0.00	-0.01	0.34	8.81**	(0,22)	9.46	0.00	14.55	0.07
Germany	0.00	0.30	0.43	5.76**	(0,22)	6.81	0.03	10.63	0.22
Japan	0.00	0.84	0.50	11.81**	(0,22)	14.94	0.00	26.59	0.00
Switzerland	0.00	0.75	0.14	3.27**	(0,22)	4.79	0.09	6.72	0.57

Note: ** and * represent 1 and 5 percent significance levels respectively

MA(22) Error Estimation



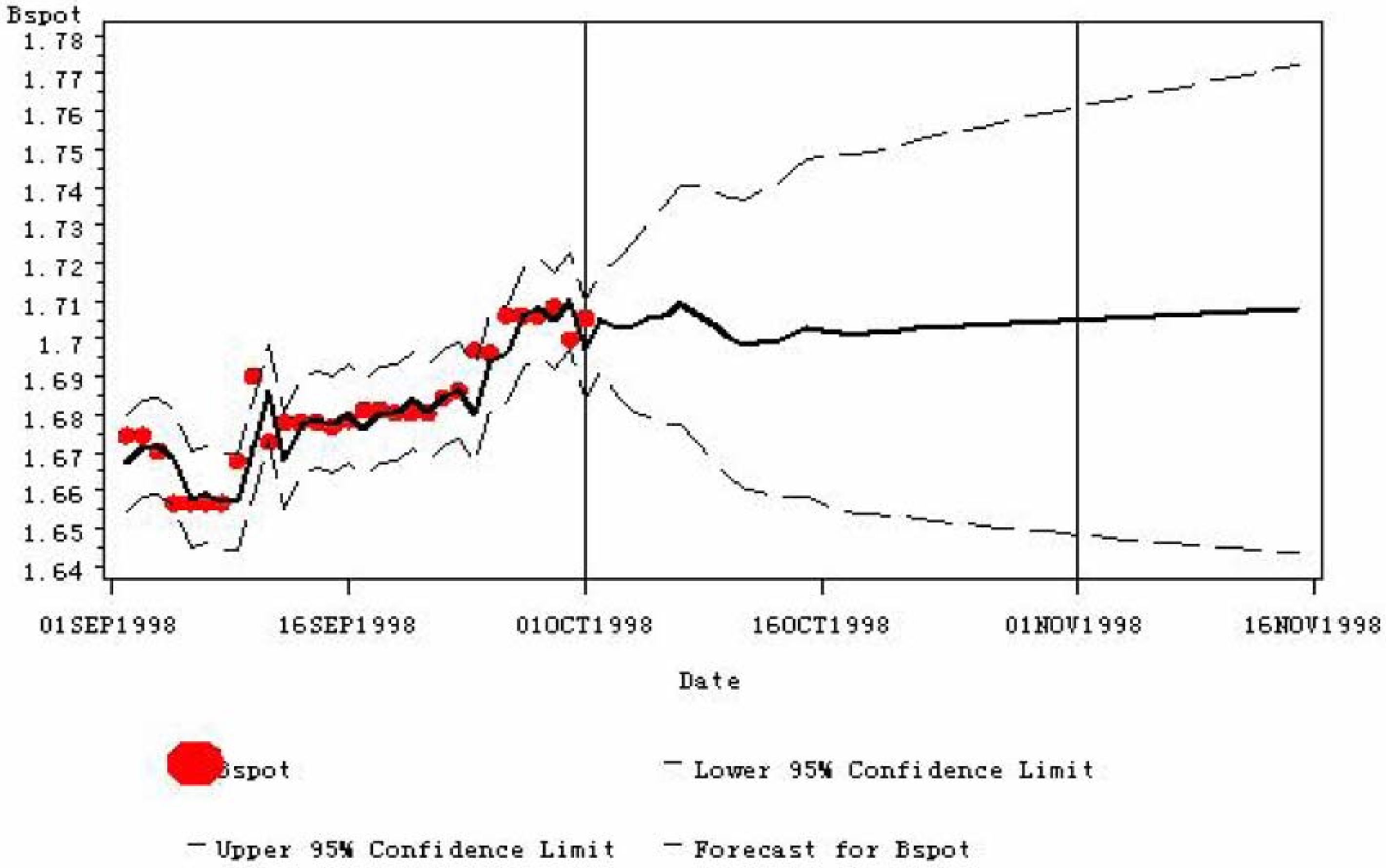


Figure 1: Forecasting Next Two Months

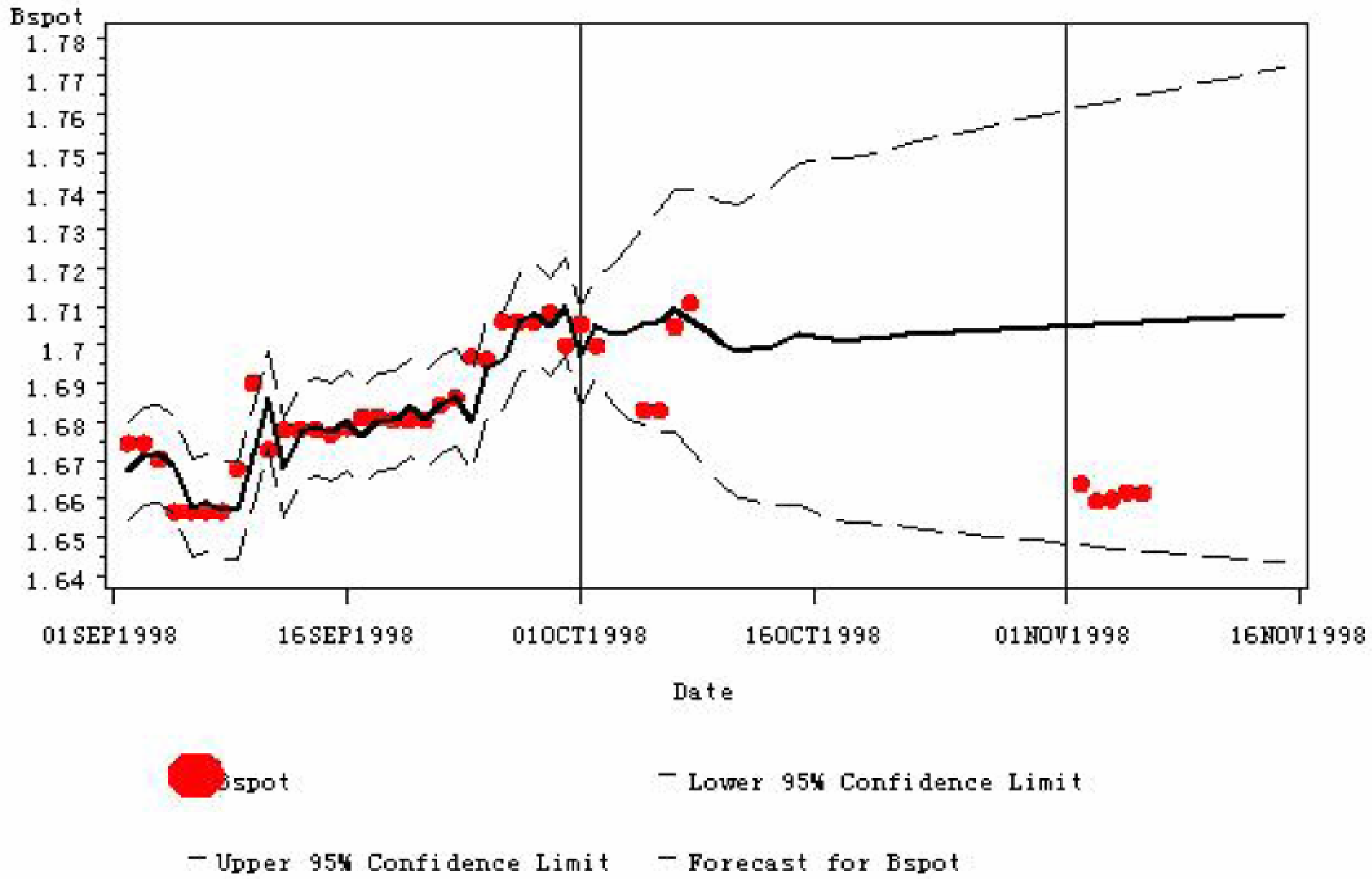


Figure 2: Forecasting the First Week of Next Two Months

Two Forecasting Senario

Forecasting Senario	02/11/98	03/11/98	04/11/98
Real Spot Rate	1.664	1.6595	1.66
Spot Rate ARIMA	1.6968	1.6970	1.6972
	(1.6411, 1.7525)	(1.6408, 1.7532)	(1.6405, 1.7539)
Model Equation (7)	1.6552	1.6442	1.6451
	(1.6387, 1.6719)	(1.6133, 1.6745)	(1.6119, 1.6790)

Forecasting Senario	05/11/98	06/11/98
Real Spot Rate	1.6618	1.6615
Spot Rate ARIMA	1.6974	1.6976
	(1.6401, 1.7547)	(1.6398, 1.7554)
Model Equation (7)	1.6461	1.6687
	(1.6106, 1.6822)	(1.6311, 1.7072)

Note: The range inside parenthesis represents lower and uper 95 percent confident limits

Unit Root Test

- Spurious Regression Problem, Granger & Newbold (1974) and Phillips (1986).

- Unit Root Test
  - Augmented Dickey-Fuller test (Dickey and Fuller, 1979)

```
proc arima data=path.yx30;
identify var=y_b  stationarity=(ADF=(1,3));
run;
```

  - Phillips-Perron Test (Phillips and Perron, 1988)

```
proc arima data=path.yx30;
identify var=y_b(1)  stationarity=(PP=5);
run;
```

Table 1

Unit root nonstationary test on spot and forward rates

Country	Spot exchange rate				Forward exchange rate			
	ADF	*p*	PP	*p*	ADF	*p*	PP	*p*
Britain	-1.77	0.40	-1.77	0.40	-1.75	0.41	-1.75	0.41
Canada	-0.97	0.77	-1.12	0.71	-1.11	-0.71	-1.18	0.69
France	-1.45	0.56	-1.44	0.56	-1.38	-0.59	-1.41	0.58
Germany	-1.37	0.60	-1.37	0.60	-1.37	0.60	-1.38	0.59
Japan	-1.23	0.66	-1.29	0.64	-1.40	0.58	-1.42	0.58
Switzerland	-1.90	0.33	-1.90	0.33	-2.03	0.27	-2.18	0.21

Note: p means p-value.

Table 2

Unit root test on depreciation rate and forward premium

Country	$s_{t+1} - s_t$				$f_{t,t+1} - s_t$			
	ADF	p	PP	p	ADF	p	PP	p
Britain	-11.17	0.00	-11.24	0.00	-17.53	0.00	-26.05	0.00
Canada	-12.09	0.00	-13.26	0.00	-28.10	-0.00	-42.99	0.00
France	-10.94	0.00	-11.21	0.00	-18.84	-0.00	-26.16	0.00
Germany	-10.92	0.00	-11.22	0.00	-19.06	0.00	-28.78	0.00
Japan	-11.10	0.00	-12.28	0.00	-36.95	0.00	-56.43	0.00
Switzerland	-11.13	0.00	-11.30	0.00	-35.36	0.00	-54.02	0.00

Note: p means p-value.

Cointegration

- Generally speaking, a linear combination of two $I(1)$ series leads to a new $I(1)$ series.
- If a linear combination of two $I(1)$ series leads to a new $I(0)$ series, the cointegration happens.
- Cointegration test
- ✓ Do the unit root test on the linear combination of two series.
- ✓ Johansen cointegration test.

```
proc varmax data=cointegration;
     model b30day b30spot /p=6 noint cointtest=(johansen);
run;
```

Cointegration Rank Test

H_0: Rank=r	H_1: Rank>r	Eigenvalue	Trace	5% Critical Value	Drift InECM	DriftIn Process
0	0	0.0654	337.98	12.21	NOINT	Constant
1	1	0.0001	0.44	4.14		

Error Correction Model

- Error Correction Model General Form:

$$\Delta x_t = m_1 + \rho_1 z_{t-1} + \gamma_i lags(\Delta x_t, \Delta y_t) + \xi_{x_t}$$

$$\Delta y_t = m_2 + \rho_2 z_{t-1} + \delta_i lags(\Delta x_t, \Delta y_t) + \xi_{y_t}$$

- $z_t = x_t - \beta y_t$ is named the error-correction term and the long-run equilibrium is represented by the coefficient β
- The deviations from z_{t-1} could be adjusted by the parameter ρ_1 and ρ_2. The signs of them tell us the moving direction of x_t and y_t for the dynamic adjustment.

ECM in the Forward Market

- Error Correction Model

$$\Delta s_{t+2} = \alpha_s + \beta_s \left(f_{t,t+1} - \beta s_{t+1} \right) + \gamma_i lags \left(\Delta s_{t+2}, \Delta f_{t+1,t+2} \right) + \varepsilon_{s_{t+2}}$$

$$\Delta f_{t+1,t+2} = \alpha_f + \beta_f \left(f_{t,t+1} - \beta s_{t+1} \right) + \delta_i lags \left(\Delta s_{t+2}, \Delta f_{t+1,t+2} \right) + \varepsilon_{f_{t+1,t+2}}$$

```
proc varmax data=cointegration;
     model forward spot / p=5 noint lagmax=6 ecm=(rank=1);
run;
```

- Threshold Error Correction Model

$$\Delta s_{t+2} = \alpha_s + \beta_s \left(f_{t,t+1} - \beta s_{t+1} - d \right) + \gamma_i lags \left(\Delta s_{t+2}, \Delta f_{t+1,t+2} \right) + \varepsilon_{s_{t+2}}$$

$$\Delta f_{t+1,t+2} = \alpha_f + \beta_f \left(f_{t,t+1} - \beta s_{t+1} - d \right) + \delta_i lags \left(\Delta s_{t+2}, \Delta f_{t+1,t+2} \right) + \varepsilon_{f_{t+1,t+2}}$$

```
proc syslin data=path.data SUR;
```

Thank You!